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DYNAMIC COMPETITIVE RESOURCE SHARING IN MULTIMEDIA NETWORKS [1]

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Abstract: In this paper, we develop a methodology for dynamic resource sharing in multimedia networks with competing traffic types. We introduce dynamic queueing models for multimedia traffic types, as well as approximate linearized dynamic queueing models.

We formulate the problem as an dynamic non-cooperative Nash game and state the conditions for non-cooperative equilibrium. Finally, we propose load sharing, routing and admission control algorithms, which state that each traffic type should be allocated only on paths and destinations that minimize its Hamiltonian.

Keywords: dynamic routing, dynamic admission control, dynamic load sharing, dynamic Nash games, dynamic non-cooperative games, multimedia networks.

1. INTRODUCTION

The integration of broadband communication networks, high performance computers, interactive TV, multimedia applications and consumer electronics is changing the way we interact. Advances in multimedia and networking technologies have fueled the rapid development of multimedia applications over broadband networks. Multimedia applications such as tele-education & tele-training, digital libraries, Internet multimedia resource discovery & retrieval, tele-working, distributed cooperative work, multimedia teleconferencing & mail, virtual reality tele-simulations, virtual community, multiplayer video games, interactive TV, video-on-demand, audio-on-demand, tele-shopping, tele-banking, tele-diagnosing & tele-medicine, tele-publishing, news-on-demand e.t.c. are driving the need to support a wide variety of advanced services over high speed broadband networks.

These distributed multimedia applications impose new traffic requirements on networking. In multimedia networks there are many traffic types that require processing and communication. These multiple traffic types have different traffic characteristics, service requirements, performance objectives and constraints.

The state of a real network is continually changing due to topology changes and real time traffic fluctuations. Therefore, the control decisions should depend on the current network state. In this paper, we consider the dynamic resource sharing problem and describe the dynamic evolution of the network state by a state space model of nonlinear difference equations. We model the problem on the path flow space and introduce dynamic

[1] The methodology proposed in this paper was first presented in A.A. Economides, *"A Unified Game-Theoretic Methodology for the Joint Load Sharing, Routing and Congestion Control Problem"*, Ph.D. Dissertation, University of Southern California, Los Angeles, August 1990.

queueing models for the number of packets both in the queue and in the system (queue plus service) for multimedia traffic types. These dynamic queueing models describe the evolution of the average number of packets in every resource for each traffic type. Furthermore, we introduce approximate linearized dynamic queueing models, in order to have a linear-quadratic problem for which there is extensive literature.

The usual approach to network design and control is the minimization of a single global cost function, possibly a combination of multiple objectives. Thus, it is assumed that all users in the network cooperate for the socially optimum, such as optimizing the average packet delay. Furthermore, previous research is primarily concentrated on networks with a single traffic type. However, in a real multimedia network, there is a diversity of users and traffic types, each with possibly different objectives and different quality of service requirements. The different decision-makers compete for the limited common resources of the network in order to optimize their own objectives, not caring for the penalty they cause to the others. For example, different telecommunication companies may share the same communication links and one of them may want to maximize its packet throughput, another may want to minimize its packet delay and a third may want to minimize its packet blocking probability. Thus, each company is seeking to optimize its own performance in competition to the other companies. In this paper, we consider the resource sharing problem in a multimedia network with multiple competitive decision-makers.

Previous studies taking a game theoretic approach to computer network resource allocation problems consider the quasi-static flow control problem [1-15] and routing problem [16-19] as a *static Nash game.* On the other hand, when the decision-makers have different rights and power, we formulated the joint load sharing, routing and admission control problem [17, 20, 21] as a *static Stackelberg game.* Furthermore, we considered the dynamic case of the problem and formulated it [17,22] as a *dynamic Stackelberg game.*

In this paper, we consider the dynamic resource sharing problem in multimedia networks with multiple competing traffic types and multiple objectives. We formulate the dynamic problem as a *dynamic non-cooperative Nash game* and derive the non-cooperative equilibrium conditions. Then we propose load sharing, admission and routing algorithms for each traffic type. Externally arriving traffic at a source is assigned to the destination for which the first derivative of its Hamiltonian with respect to (w.r.t.) its corresponding load sharing fraction is minimum. However, this traffic may be rejected if the first derivative of its Hamiltonian w.r.t. its admission fraction is less than the first derivative of its Hamiltonian w.r.t. its routing fractions to any path to its destination. If it is accepted, then it is routed to its destination via the path that has the minimum first derivative of its Hamiltonian w.r.t. its corresponding routing fraction.

2. DYNAMIC MODELS FOR MULTIMEDIA NETWORKS

In this section, we model the dynamic resource sharing problem in multimedia networks with competing traffic types. We also introduce dynamic queueing models to describe the dynamic evolution of the network state. The general structure of these dynamic models is based on the idea that the number of packets in a resource increases by the number of packet arrivals to and decreases by the number of packet departures from that resource during a time interval.

Traffic type c, requiring processing anywhere in the network, arrives at the source node $[s.]$ with instantaneous at time t external arrival rate $\gamma^c_{[s.]}(t, \mathbf{N}(t)) \geq 0$. Note, that the external arrival rate depends on the total traffic currently in the network ($\mathbf{N}(t) \geq \mathbf{0}$: vector of the average number of packets at all network resources). This means that when the traffic in the network is heavy, some users may be discouraged from submitting traffic into the network and possibly postpone it for later. On the other hand, when the network is lightly loaded, this may motivate some users in using the network. A load sharing decision is made as to where this traffic will be processed. Let $\psi^c_{[sd]}(t) \geq 0$ be the fraction of this traffic sent for processing to node $[.d]$. Since only one destination node is selected, the sum of the load sharing fractions from node $[s.]$ to all of its possible destinations $[.d]$ is equal to one: $\sum\limits_{[.d]} \psi^c_{[sd]}(t) = 1$. Let $\mathbf{LS}^c$ be the load sharing constraint set for traffic type c, which includes all these fractions with their constraints for all sources.

Since packets arrive at the source node $[s.]$ with external arrival rate $\gamma^c_{[s.]}(t, \mathbf{N}(t))$, then the traffic from source node $[s.]$, transferred for processing to the destination node $[.d]$, will be $\gamma^c_{[s.]}(t, \mathbf{N}(t)) * \psi^c_{[sd]}(t)$.

Therefore, the instantaneous at time t total traffic type c that is transferred for processing to the destination node $[.d]$, due to load sharing is:

$$\lambda^c_{[.d]}(t) = \sum_{[s.]} \gamma^c_{[s.]}(t, \mathbf{N}(t)) * \psi^c_{[sd]}(t)$$

After deciding which fraction of the traffic will be processed by destination node $[.d]$, it should be transferred there through a route. For routing, we must specify which path between source-destination $[sd]$ will be selected. In addition, some packets may be rejected outside of the network, for congestion control reasons. So, let the fraction of rejected traffic for the source-destination $[sd]$ be $\phi^c_{o[sd]}(t) \geq 0$, and the fraction of traffic routed through path $\pi[sd]$ be $\phi^c_{\pi[sd]}(t) \geq 0$. Since the traffic for the source-destination $[sd]$ may only be rejected or routed through a path, all these fractions must sum to one: $\phi^c_{o[sd]}(t) + \sum\limits_{\pi[sd]} \phi^c_{\pi[sd]}(t) = 1$. Let $\mathbf{AR}^c$ be the admission and routing constraint set for traffic type c, which includes all these fractions with their constraints for all source-destinations.

Furthermore, traffic arrives at the source node $[s.]$ and requires only transfer to the destination node $[.d]$ (i.e. no processing), with instantaneous at time t external arrival rate $\gamma^c_{[sd]}(t, \mathbf{N}(t)) \geq 0$. Again, this external arrival rate depends on the total traffic currently in the network. Since there is traffic $\gamma^c_{[s.]}(t, \mathbf{N}(t)) * \psi^c_{[sd]}(t)$ (due to load sharing decisions) and traffic $\gamma^c_{[sd]}(t, \mathbf{N}(t))$ (due to communication requirements) for the source-destination $[sd]$, the total traffic type c for the source-destination $[sd]$ is $\gamma^c_{[sd]}(t, \mathbf{N}(t)) + \gamma^c_{[s.]}(t, \mathbf{N}(t)) * \psi^c_{[sd]}(t)$.

Due to the admission control, the fraction of this traffic that will be rejected is $\phi^c_{o[sd]}(t)$. So, let the instantaneous at time t rejected traffic for the source-destination $[sd]$ be

$$\lambda^c_{o[sd]}(t) = (\gamma^c_{[sd]}(t, \mathbf{N}(t)) + \gamma^c_{[s.]}(t, \mathbf{N}(t)) * \psi^c_{[sd]}(t)) * \phi^c_{o[sd]}(t)$$

Another fraction $\phi^c_{\pi[sd]}(t)$ of this traffic will be routed through path $\pi[sd]$. Therefore, the resulting traffic on path $\pi[sd]$ will be $(\gamma^c_{[sd]}(t, \mathbf{N}(t)) + \gamma^c_{[s.]}(t, \mathbf{N}(t)) * \psi^c_{[sd]}(t)) * \phi^c_{\pi[sd]}(t)$. This traffic will be assigned to the links and nodes that constitute this path. So, the instantaneous at time t incoming traffic type c on link ij will be the sum of all path traffic type c that traverse this link :

$$\lambda_{ij}^c(t) = \sum_{[sd]} \sum_{\pi[sd]} (\gamma_{[sd]}^c(t, \mathbf{N}(t)) + \gamma_{[s.]}^c(t, \mathbf{N}(t)) * \psi_{[sd]}^c(t)) * \phi_{\pi[sd]}^c(t) * 1_{ij \in \pi[sd]}(t)$$

where $1_{ij \in \pi[sd]}(t)$ is the indicator function that link ij is on the path $\pi[sd]$.

Similarly, the instantaneous at time t incoming traffic type c at node i will be the sum of all path traffic type c that traverse this node :

$$\lambda_i^c(t) = \sum_{[sd]} \sum_{\pi[sd]} (\gamma_{[sd]}^c(t, \mathbf{N}(t)) + \gamma_{[s.]}^c(t, \mathbf{N}(t)) * \psi_{[sd]}^c(t)) * \phi_{\pi[sd]}^c(t) * 1_{i \in \pi[sd]}(t)$$

Next, we describe the dynamic evolution of the network state using a state space model for each network resource. The real network state is a discrete-state stochastic process. However, the decision makers cannot have instantaneous knowledge of the global state at every instant. So, even if we solve the stochastic problem, it will be difficult to implement the solution. Therefore, we use the deterministic approximation of this stochastic process by its expected value. We define as state of a network resource, the average number of packets of all traffic types at this resource (non-negative continuous-state continuous-time process). Note, that we may also describe the network state in a more detailed way using for example both the packet and the call processes at each resource [17,23,24]. The average number of packets at a resource increases during a time interval by the average number of packets that arrive during this time interval and decreases by the average number of packets that depart during this interval. So, let $N_{ij}^c(t) \geq 0$ be the average number of traffic type c packets at resource ij at time t and $\lambda_{ij}^c(t, \mathbf{N}(t), \mathbf{\Phi}^c(t), \mathbf{\Psi}^c(t)) \geq 0$ be the instantaneous at time t incoming traffic (arrival rate) type c at resource ij. The service scheduling employed by the resource ij and the service requirement distributions of the various traffic type packets determine this instantaneous at time t outcoming traffic (departure rate) type c at resource ij, $d_{ij}^c(t, \mathbf{N}_{ij}(t)) \geq 0$, which is a function of the average number of packets of all traffic types at the resource ij, $\mathbf{N}_{ij}(t) = [N_{ij}^1(t), ..., N_{ij}^c(t), ..., N_{ij}^C(t)]$, with $d_{ij}^c(t, \mathbf{0}) = 0$. Then we can write

$$\dot{N}_{ij}^c(t) = \lambda_{ij}^c(t, \mathbf{N}(t), \mathbf{\Phi}^c(t), \mathbf{\Psi}^c(t)) - d_{ij}^c(t, \mathbf{N}_{ij}(t))$$

For the whole network, let $\mathbf{N}(t) \geq 0$ be the vector network state, $\mathbf{\Lambda}(t, \mathbf{N}(t), \mathbf{\Phi}(t), \mathbf{\Psi}(t)) \geq \mathbf{0}$ be the vector arrival rate and $\mathbf{D}(t, \mathbf{N}(t)) \geq \mathbf{0}$ be the vector departure rate at time t, we can write

$$\dot{\mathbf{N}}(t) = \mathbf{\Lambda}(t, \mathbf{N}(t), \mathbf{\Phi}(t), \mathbf{\Psi}(t)) - \mathbf{D}(t, \mathbf{N}(t))$$

Note again that in this general model, the arrival rate depends on the traffic already in the network. Under specific assumptions on the network operation and the traffic distributions, the above abstract form of the state space model reduces to specific differential equations. The arrival rate and the departure rate from a resource should be nonnegative, nondecreasing, continuous and differential functions of the number of packets there. It is also desirable, the arrival rate to be a convex function, while the departure rate to be a concave function of the number of packets there. We can write such differential equations for each traffic type c at each network resource. Then, a differential vector equation describes the dynamic network evolution

$$\dot{\mathbf{N}}(t) = \mathbf{f}(t, \mathbf{N}(t), \mathbf{\Phi}(t), \mathbf{\Psi}(t))$$

In the next paragraphs, we introduce dynamic models for multimedia traffic. In these models, the arrival rate at a resource is independent of the current traffic in the resource. We encourage research on deriving dynamic queueing models with arrival rates depending on the number of packets currently in the queue or in the system.

2.1. Dynamic M/G/1 Queueing Models

Here, we introduce dynamic queueing models for multiple class $M/G/1$ queues. Let packets with mean service requirement $1/\mu$ and second moment of service requirement $\overline{x^2}$ arrive with Poisson distribution at a resource with service rate at time t, $C(t)$. Then, the average number of class c packets in $M/G/1$ queues is given by

$$N^c = \rho^c + \rho^c * \frac{\rho * \overline{x^2} * \mu^2}{2(1-\rho)} \quad \forall\ c$$

where ρ^c is the utilization for class c and $\rho = \sum_c \rho^c$ is the overall utilization.

Solving the above system of equations for ρ^c, we have the utilization for class c as a function of the average number of packets for each class:

$$\rho^c = \frac{2N^c * \left(1 - \overline{x^2} * \mu^2 - \sum_k N^k + \sqrt{\left(1 + \sum_k N^k\right)^2 - 2\sum_k N^k * (2 - \overline{x^2} * \mu^2)}\right)}{(2 - \overline{x^2} * \mu^2) * \left(1 - \sum_k N^k + \sqrt{\left(1 + \sum_k N^k\right)^2 - 2\sum_k N^k * (2 - \overline{x^2} * \mu^2)}\right)}$$

Then we propose the following dynamic model for multiple class $M/G/1$ queues [17]:

$$\dot{N}^c(t) = \lambda^c(t) - \mu C(t) * \frac{2N^c(t)}{2 - \overline{x^2} * \mu^2} *$$

$$* \frac{\left(1 - \overline{x^2} * \mu^2 - \sum_k N^k(t) + \sqrt{\left(1 + \sum_k N^k(t)\right)^2 - 2\sum_k N^k(t) * (2 - \overline{x^2} * \mu^2)}\right)}{\left(1 - \sum_k N^k(t) + \sqrt{\left(1 + \sum_k N^k(t)\right)^2 - 2\sum_k N^k(t) * (2 - \overline{x^2} * \mu^2)}\right)}$$

For exponential service, general service with Processor Sharing (*P.S.*) discipline and deterministic service times, the above model gives the following dynamic models:

$$
\begin{aligned}
\dot{N}^c(t) &= \lambda^c(t) - \mu C(t) * \frac{N^c(t)}{1 + \sum\limits_k N^k(t)} && M/M/1 \\
\dot{N}^c(t) &= \lambda^c(t) - \mu C(t) * \frac{w^c * N^c(t)}{1 + \sum\limits_k w^k * N^k(t)} && \textit{class discriminating P.S.}
\end{aligned}
$$

$$
\dot{N}^c(t) = \lambda^c(t) - \mu C(t) * \frac{2N^c(t) * \left(- \sum\limits_k N^k(t) + \sqrt{1 + \left(\sum\limits_k N^k(t) \right)^2} \right)}{1 - \sum\limits_k N^k(t) + \sqrt{1 + \left(\sum\limits_k N^k(t) \right)^2}} \quad M/D/1
$$

Also, for multiple class $M/M/\infty$ queues we have the following dynamic model:

$$
\dot{N}^c(t) = \lambda^c(t) - \mu^c C(t) * N^c(t) \quad M/M/\infty
$$

Considering other more complicated queueing models, one may derive dynamic queueing models for other cases.

2.2. Dynamic Queueing Models for the Packets in Queue

In future high speed networks, we will have information only about the average number of packets in the queue (not both in the queue and in service), due to the enormous number of packets that will be in transit into the network. Therefore, it is also useful to have dynamic queueing models for the average number of packets in the queue. Here, we present the methodology of deriving such models for the case of multiple class $M/G/1$ queues. We encourage research on more complicated queueing models with multiple servers and queues, various arrival distributions, service scheduling e.t.c. The average number of class c packets in queue for a multiple class $M/G/1$ queue is given by

$$
N_Q^c = \rho^c * \frac{\rho * \overline{x^2} * \mu^2}{2(1-\rho)} \quad \forall\ c
$$

Solving the above system of equations, we have the utilization for class c, ρ^c, as a function of the average number of packets in queue for all classes

$$
\rho^c = \frac{2N_Q^c * \left(\overline{x^2} * \mu^2 + \sum\limits_k N_Q^k - \sqrt{\left(\sum\limits_k N_Q^k \right)^2 + 2 \sum\limits_k N^Q * \overline{x^2} * \mu^2} \right)}{\overline{x^2} * \mu^2 * \left(- \sum\limits_k N_Q^k + \sqrt{\left(\sum\limits_k N_Q^k \right)^2 + 2 \sum\limits_k N^Q * \overline{x^2} * \mu^2} \right)}
$$

Then we propose the following dynamic model for multiple class $M/G/1$ queues [17]:

$$\dot{N}_Q^c(t) = \lambda^c(t) - \mu C(t) * \frac{2N_Q^c(t)}{\overline{x^2} * \mu^2} *$$

$$* \frac{\overline{x^2} * \mu^2 + \sum_k N_Q^k - \sqrt{\left(\sum_k N_Q^k\right)^2 + 2\sum_k N^Q * \overline{x^2} * \mu^2}}{-\sum_k N_Q^k + \sqrt{\left(\sum_k N_Q^k\right)^2 + 2\sum_k N^Q * \overline{x^2} * \mu^2}}$$

For exponential service, general service with Processor Sharing ($P.S.$) discipline and deterministic service time, the above model gives the following dynamic models:

$$\dot{N}_Q^c(t) = \lambda^c(t) - \mu C(t) * N_Q^c(t) *$$

$$* \frac{2 + \sum_k N_Q^k - \sqrt{\left(\sum_k N_Q^k\right)^2 + 4\sum_k N^Q}}{-\sum_k N_Q^k + \sqrt{\left(\sum_k N_Q^k\right)^2 + 4\sum_k N^Q}} \quad M/M/1 \text{ } or \text{ } P.S.$$

$$\dot{N}_Q^c(t) = \lambda^c(t) - \mu C(t) * 2N_Q^c(t) *$$

$$* \frac{1 + \sum_k N_Q^k - \sqrt{\left(\sum_k N_Q^k\right)^2 + 2\sum_k N^Q}}{-\sum_k N_Q^k + \sqrt{\left(\sum_k N_Q^k\right)^2 + 2\sum_k N^Q}} \quad M/D/1$$

Note that for single class, we have:

$$\begin{aligned}
\dot{N}_Q(t) &= \lambda(t) - \mu C(t) * \frac{-N_Q(t) + \sqrt{(N_Q)^2 + 2N_Q * \overline{x^2} * \mu^2}}{\overline{x^2} * \mu^2} && M/G/1 \\
\dot{N}_Q(t) &= \lambda(t) - \mu C(t) * \frac{-N_Q(t) + \sqrt{(N_Q)^2 + 4N_Q}}{2} && M/M/1 \text{ } or \text{ } P.S. \\
\dot{N}_Q(t) &= \lambda(t) - \mu C(t) * \left(-N_Q(t) + \sqrt{(N_Q)^2 + 2N_Q}\right) && M/D/1
\end{aligned}$$

2.3. Linearized Dynamic Queueing Models

Although the dynamic queueing models describe accurately the dynamic behavior of the queue, they depend nonlinearly on the average number of packets in the system (except the $M/M/\infty$ model). Therefore the analytical solution of the dynamic optimization problem usually becomes intractable. Next, we propose the linearization of dynamic queueing models, that gives simpler models. For example, the linearized multiple class $M/M/1$ queueing model is the following:

$$
\begin{aligned}
\dot{N}^c(t) &= \lambda^c(t) - \mu C * \frac{N^c(t)}{1+\sum\limits_k N^k(t)} \\
&\approx \lambda^c(t) - \mu C * \frac{\overline{N^c}}{1+\sum\limits_k \overline{N^k}} - \mu C * \sum_k \frac{\partial}{\partial \overline{N^k}} \left(\frac{\overline{N^c}}{1+\sum\limits_k \overline{N^k}} \right) * (N^k(t) - \overline{N^k}) \\
&\approx \lambda^c(t) - \mu C * \frac{\overline{N^c}}{1+\sum\limits_k \overline{N^k}} - \mu C * \frac{1+\sum\limits_{k \neq c} \overline{N^k}}{\left(1+\sum\limits_k \overline{N^k}\right)^2} * (N^c(t) - \overline{N^c}) \\
&\quad + \mu C * \sum_k \frac{N^c}{\left(1+\sum\limits_n \overline{N^n}\right)^2} * (N^k(t) - \overline{N^k})
\end{aligned}
$$

$$\approx \quad \lambda^c(t) - \mu C * \frac{\dfrac{\lambda^c}{\mu C - \sum_k \lambda^k}}{1 + \dfrac{\sum_k \lambda^k}{\mu C - \sum_k \lambda^k}}$$

$$-\mu C * \frac{1}{1 + \dfrac{\sum_k \lambda^k}{\mu C - \sum_k \lambda^k}} * \left(N^c(t) - \frac{\lambda^c}{\mu C - \sum_k \lambda^k} \right)$$

$$+\mu C * \frac{\lambda^c}{\mu C \sum_n \lambda^n} * \sum_k \frac{1}{\left(1 + \dfrac{\sum_n \lambda^n}{\mu C - \sum_n \lambda^n} \right)^2} * \left(N^k(t) - \frac{\lambda^k}{\mu C - \sum_n \lambda^n} \right)$$

Finally, we have the following linearized model for multi-class $M/M/1$ queues [17]:

$$\dot{N}^c(t) \approx \lambda^c(t) - \lambda^c * \frac{\sum_k \lambda^k}{\mu C} - (\mu C - \sum_k \lambda^k) * N^c(t) + \frac{\lambda^c * \mu C - \sum_n \lambda^n}{\mu C} * \sum_k N^k(t)$$

The above model satisfies the steady-state flow conservation

$$\lambda^c - \lambda^c * \frac{\sum_k \lambda^k}{\mu C} = (\mu C - \sum_k \lambda^k) * \overline{N^c} - \frac{\lambda^c * \mu C - \sum_n \lambda^n}{\mu C} * \sum_k \overline{N^k} \Leftrightarrow \overline{N^c} = \frac{\lambda^c}{\mu C - \sum_k \lambda^k}$$

Similarly, we may derive dynamic models for the average number of packets in the system (queue plus service), or in the queue for other queueing models. We encourage research on queues with general arrival and service distributions, multiple queues, various service policies e.t.c.

Another approximate model for multiple class $M/M/1$ queues is the following [17]:

$$\begin{aligned}
\dot{N}^c(t) &= \lambda^c(t) - \mu C * \frac{N^c(t)}{1 + \sum\limits_k N^k(t)} \\
&\approx \lambda^c(t) - \mu C * \frac{1}{1 + \sum\limits_k \overline{N^k}} * N^c(t) \\
&\approx \lambda^c(t) - \mu C * \frac{1}{1 + \dfrac{\sum\limits_k \lambda^k}{\mu C - \sum\limits_k \lambda^k}} * N^c(t) \\
&\approx \lambda^c(t) - (\mu C - \sum_k \lambda^k) * N^c(t)
\end{aligned}$$

The above model also satisfies the steady-state flow conservation

$$\lambda^c = (\mu C - \sum_k \lambda^k) * N^c \Leftrightarrow N^c = \frac{\lambda}{\mu C - \sum\limits_k \lambda^k}$$

For this particular model, the inverse of the first derivative of the departure rate with respect to the average number of packets becomes:

$$\begin{aligned}
\left[\frac{\partial}{\partial N^c}\left((\mu C - \sum_k \lambda^k) * N^c\right)\right]^{-1} &= \frac{1}{\mu C - \sum\limits_k \lambda^k} = \frac{\frac{1}{\mu C}}{1 - \frac{\sum\limits_k \lambda^k}{\mu C}} \\
&= \frac{\frac{1}{\mu C}}{1 - \dfrac{\sum\limits_k N^k}{1 + \sum\limits_k N^k}} = \frac{1 + \sum\limits_k N^k}{\mu C}
\end{aligned}$$

This result explains why the shortest route routing achieves good performance [17].

After the model linearization, the system state is described by the following state equation

$$\dot{\mathbf{N}} = \mathbf{A} * \mathbf{N} + \mathbf{B} * (\mathbf{\Phi}, \mathbf{\Psi}) \qquad \mathbf{N}_0 : given$$

with cost function

$$\int_{t_0}^{t_f} \frac{1}{2} * (\mathbf{N}^T * \mathbf{S} * \mathbf{N} + (\mathbf{\Phi}, \mathbf{\Psi})^T * \mathbf{R} * (\mathbf{\Phi}, \mathbf{\Psi}))dt$$

where $\mathbf{A}, \mathbf{B}, \mathbf{S}, \mathbf{R}$ are suitable matrices. Thus, we can use results from the optimal control theory on linear-quadratic problems, to solve the resource sharing problem.

2.4. Cost Functions

In this section, we introduce cost functions that can be used in the dynamic problem. Let $[\mathbf{\Phi}^c, \mathbf{\Psi}^c]$ and $[\mathbf{\Phi}, \mathbf{\Psi}]$, be the strategy of traffic type c and of all traffic types, respectively, during the whole duration of the problem. Define the instantaneous at time t cost function for traffic type c to be $g^c(t, \mathbf{N}(t), \mathbf{\Phi}(t), \mathbf{\Psi}(t))$, and the total cost function during the whole duration of the problem, from the initial time t_0 to the final time t_f, becomes

$$J^c(\mathbf{\Phi}, \mathbf{\Psi}) = \int_{t_0}^{t_f} g^c(t, \mathbf{N}(t), \mathbf{\Phi}(t), \mathbf{\Psi}(t))dt$$

For the infinite horizon problem, we consider the following cost function

$$J^c(\mathbf{\Phi}, \mathbf{\Psi}) = \int_{t_0}^{\infty} e^{-kt} g^c(t, \mathbf{N}(t), \mathbf{\Phi}(t), \mathbf{\Psi}(t))dt$$

where k is a discount cost.

We can decompose this cost as the sum of its average cost at every network resource minus its benefit from operating the network. Here, we examine the most general case where the cost function at a resource may depend on the traffic over the whole network. Desired properties of a cost function are to be nonnegative, bounded from above, nondecreasing, continuous, differentiable and convex $\forall N_i^c(t) \geq 0$.

In the optimization problem, we shall use the Hamiltonian and Lagrangian functions. Let the Hamiltonian for the traffic type c be:

$$H^c(t, \mathbf{N}, \mathbf{\Phi}, \mathbf{\Psi}, \mathbf{P}^c) = g^c(t, \mathbf{N}, \mathbf{\Phi}, \mathbf{\Psi}) + \mathbf{P}^c * \mathbf{f}(t, \mathbf{N}, \mathbf{\Phi}, \mathbf{\Psi})$$

where $\mathbf{P}^c = [\dots\ P_{ij}^{c,k}\ \dots\ P_i^{c,k}\ \dots\ P_{o[sd]}^{c,k}\ \dots\ P_{[.d]}^{c,k}\ \dots]$: vector of costate variables associated with the state equations.

Let also the Lagrangian for the traffic type c be:

$$\begin{aligned} L^c(t, \mathbf{N}, \mathbf{\Phi}, \mathbf{\Psi}, \mathbf{P}^c, \mathbf{Q}^c) &= H^c(t, \mathbf{N}, \mathbf{\Phi}, \mathbf{\Psi}, \mathbf{P}^c) + \sum_{[s.]} Q_{[s.]}^c * \left[1 - \sum_{[.d]} \psi_{[sd]}^c\right] \\ &+ \sum_{[sd]} Q_{[sd]}^c * \left[1 - \phi_{o[sd]}^c - \sum_{\pi[sd]} \phi_{\pi[sd]}^c\right] \end{aligned}$$

with $\phi_{o[sd]}^c,\ \phi_{\pi[sd]}^c,\ \psi_{[sd]}^c \geq 0 \quad \forall\ \pi[sd],\ [sd],\ c,$

where $\mathbf{Q}^c = [... \ Q^c_{[sd]} \ ... \ Q^c_{[s.]} \ ...]$: vector of the multipliers associated with the constraints of the admission, routing and load sharing fractions.

3. NON-COOPERATIVE EQUILIBRIUM SOLUTION

In this section, we formulate the dynamic resource sharing problem on the path flow space as a dynamic non-cooperative Nash game among competing traffic types.

Packets of each traffic type try to use the resources of the multimedia network for their own benefit, ignoring the inconvenience that they cause to packets from other traffic types. Since the behavior of each traffic type is similar to that of any other traffic type, i.e. to operate optimally for its packets, next we consider packets only from traffic type c, and the effect of packets from other traffic types on them. When the traffic types are in equilibrium, no traffic type can decrease its cost by altering its decision unilaterally.

Theorem 1:

Consider the dynamic resource sharing problem in multimedia networks with C competing traffic types, with fixed initial time t_0 and final time t_f. If for each traffic type c, $H^c(t, \mathbf{N}, \mathbf{\Phi}, \mathbf{\Psi}, \mathbf{P}(t))$ is differentiable and convex in $(\mathbf{N}, \mathbf{\Phi}^c, \mathbf{\Psi}^c) \in (\mathbf{R}, \mathbf{AR}^c, \mathbf{LS}^c) \quad \forall \ t \in [t_0, t_f]$, for each fixed value of $(\mathbf{\Phi}^1, \mathbf{\Psi}^1, ..., \mathbf{\Phi}^{c-1}, \mathbf{\Psi}^{c-1}, \mathbf{\Phi}^{c+1}, \mathbf{\Psi}^{c+1}, ..., \mathbf{\Phi}^C, \mathbf{\Psi}^C)$ $\in (\mathbf{AR}^1, \mathbf{LS}^1, ..., \mathbf{AR}^{c-1}, \mathbf{LS}^{c-1}, \mathbf{AR}^{c+1}, \mathbf{LS}^{c+1}, ..., \mathbf{AR}^C, \mathbf{LS}^C)$,

then $(\mathbf{\Phi}^*(t), \mathbf{\Psi}^*(t)) \in (\mathbf{AR}, \mathbf{LS})$ is a Nash equilibrium if and only if it solves the following <u>Optimal Control Problem</u> $\forall \ t \in [t_0, t_f]$:

$$
\begin{array}{ll}
\forall \ c & \\
\textit{minimize} & \displaystyle\int_{t_0}^{t_f} g^c(t, \mathbf{N}(t), \mathbf{\Phi}^{1*}(t), \mathbf{\Psi}^{1*}(t), ..., \mathbf{\Phi}^c(t), \mathbf{\Psi}^c(t), ..., \mathbf{\Phi}^{C*}(t), \mathbf{\Psi}^{C*}(t))dt \\
\textit{with respect to} & (\mathbf{\Phi}^c(t), \mathbf{\Psi}^c(t)) \\
\textit{such that} & \dot{\mathbf{N}}(t) = \mathbf{f}(t, \mathbf{N}(t), \mathbf{\Phi}(t), \mathbf{\Psi}(t)) \\
& \mathbf{N}(t_0) = \mathbf{N}_0 \\
& (\mathbf{\Phi}^c(t), \mathbf{\Psi}^c(t)) \in (\mathbf{AR}^c, \mathbf{LS}^c)
\end{array}
$$

Proof: The proof follows from the definition of a Nash equilibrium [25,26,27]. □

Theorem 2:

Consider the dynamic resource sharing problem in multimedia networks with C competing traffic types, with fixed initial time t_0 and final time t_f. Let for each traffic type c, $g^c(t, \mathbf{N}, \mathbf{\Phi}, \mathbf{\Psi})$, $\mathbf{f}(t, \mathbf{N}, \mathbf{\Phi}, \mathbf{\Psi})$, are continuously differentiable with respect to $\mathbf{N} \in \mathbf{R}^n$, $\forall \ t \in [t_0, t_f]$. If $(\hat{\mathbf{\Phi}}^*(t, \mathbf{N}_0), \hat{\mathbf{\Psi}}^*(t, \mathbf{N}_0)) = (\mathbf{\Phi}^*(t), \mathbf{\Psi}^*(t)) \in (\mathbf{AR}, \mathbf{LS})$ is an <u>open-loop Nash equilibrium</u> and $\{\mathbf{N}^*(t), \ t \in [t_0, t_f]\}$ is the corresponding state trajectory, then $\exists \ \mathbf{P}^c(t) : [t_0, t_f] \rightarrow \mathbf{R}^n$, $\forall \ c$ continuous and piecewise continuously differentiable vector functions, such that $\forall \ t \in [t_0, t_f]$:

$$\dot{\mathbf{N}}^*(t) = \mathbf{f}(t, \mathbf{N}^*(t), \mathbf{\Phi}^*(t), \mathbf{\Psi}^*(t)) \tag{1}$$

$$\mathbf{N}^*(t_0) = \mathbf{N}_0 \tag{2}$$

$$\left[\frac{\partial H^{c*}}{\partial \phi^c_{o[sd]}} - Q^c_{[sd]}(t)\right] * \phi^{c*}_{o[sd]}(t) = 0 \qquad \forall\ [sd],\ c$$

$$\left[\frac{\partial H^{c*}}{\partial \phi^c_{\pi[sd]}} - Q^c_{[sd]}(t)\right] * \phi^{c*}_{\pi[sd]}(t) = 0 \qquad \forall\ \pi[sd],\ [sd], c$$

$$\left[\frac{\partial H^{c*}}{\partial \psi^c_{[sd]}} - Q^c_{[s.]}(t)\right] * \psi^{c*}_{[sd]}(t) = 0 \qquad \forall\ [.d],\ [s.],\ c$$

$$\frac{\partial H^{c*}}{\partial \phi^c_{o[sd]}} - Q^c_{[sd]}(t) \geq 0 \qquad \forall\ [sd],\ c$$

$$\frac{\partial H^{c*}}{\partial \phi^c_{\pi[sd]}} - Q^c_{[sd]}(t) \geq 0 \qquad \forall\ \pi[sd],\ [sd],\ c$$

$$\frac{\partial H^{c*}}{\partial \psi^c_{[sd]}} - Q^c_{[s.]}(t) \geq 0 \qquad \forall\ [.d],\ [s.],\ c$$

$$\dot{\mathbf{P}}^c(t) = -\nabla_{\mathbf{N}} H^c(t, \mathbf{N}^*, \mathbf{\Phi}^*(t), \mathbf{\Psi}^*(t), \mathbf{P}^c(t)) \quad \forall\ c \tag{3}$$

$$\mathbf{P}^c(t_f) = \mathbf{0} \qquad \forall\ c \tag{4}$$

$$\phi^{c*}_{o[sd]}(t) + \sum_{\pi[sd]} \phi^{c*}_{\pi[sd]}(t) = 1 \qquad \forall\ [sd],\ c$$

$$\sum_{[.d]} \psi^{c*}_{[sd]}(t) = 1 \qquad \forall\ [s.],\ c$$

$$\phi^{c*}_{o[sd]}(t),\ \phi^{c*}_{\pi[sd]}(t) \geq 0 \qquad \forall\ \pi[sd],\ [sd],\ c$$

$$\psi^{c*}_{[sd]}(t) \geq 0 \qquad \forall\ [.d],\ [s.],\ c$$

Proof: The Lagrangian for each traffic type c is

$$L^c = H^c + \sum_{[sd]} Q^c_{[sd]} * \left[1 - \phi^c_{o[sd]} - \sum_{\pi[sd]} \phi^c_{\pi[sd]}\right] + \sum_{[s.]} Q^c_{[s.]} * \left[1 - \sum_{[.d]} \psi^c_{[sd]}\right]$$

with $\phi^c_{o[sd]},\ \phi^c_{\pi[sd]},\ \psi^c_{[sd]} \geq 0 \quad \forall\ \pi[sd],\ [sd],\ c$

Pontryagin's maximum principle necessary conditions become:

$$\dot{\mathbf{N}}^*(t) = \mathbf{f}(t, \mathbf{N}^*(t), \mathbf{\Phi}^*(t), \mathbf{\Psi}^*(t))$$

$$\mathbf{N}^*(t_0) = \mathbf{N}_0$$

$$\frac{\partial L^{c*}}{\partial \phi^c_{o[sd]}} * \phi^{c*}_{o[sd]}(t) = 0 \;\Rightarrow\; [\frac{\partial H^{c*}}{\partial \phi^c_{o[sd]}} - Q^c_{[sd]}(t)] * \phi^{c*}_{o[sd]}(t) = 0 \quad \forall\ [sd],\ c$$

$$\frac{\partial L^{c*}}{\partial \phi^c_{\pi[sd]}} * \phi^{c*}_{\pi[sd]}(t) = 0 \;\Rightarrow\; [\frac{\partial H^{c*}}{\partial \phi^c_{\pi[sd]}} - Q^c_{[sd]}(t)] * \phi^{c*}_{\pi[sd]}(t) = 0 \;\; \forall\ \pi[sd],\ [sd], c$$

$$\frac{\partial L^{c*}}{\partial \psi^c_{[sd]}} * \psi^{c*}_{[sd]}(t) = 0 \;\Rightarrow\; [\frac{\partial H^{c*}}{\partial \psi^c_{[sd]}} - Q^c_{[s.]}(t)] * \psi^{c*}_{[sd]}(t) = 0 \quad \forall\ [.d],\ [s.],\ c$$

$$\frac{\partial L^{c*}}{\partial \phi^c_{o[sd]}} \geq 0 \;\Rightarrow\; \frac{\partial H^{c*}}{\partial \phi^c_{o[sd]}} - Q^c_{[sd]}(t) \geq 0 \quad \forall\ [sd],\ c$$

$$\frac{\partial L^{c*}}{\partial \phi^c_{\pi[sd]}} \geq 0 \;\Rightarrow\; \frac{\partial H^{c*}}{\partial \phi^c_{\pi[sd]}} - Q^c_{[sd]}(t) \geq 0 \quad \forall\ \pi[sd],\ [sd],\ c$$

$$\frac{\partial L^{c*}}{\partial \psi^c_{[sd]}} \geq 0 \;\Rightarrow\; \frac{\partial H^{c*}}{\partial \psi^c_{[sd]}} - Q^c_{[s.]}(t) \geq 0 \qquad \forall\ [.d],\ [s.],\ c$$

$$\dot{\mathbf{P}}^c(t) = -\nabla_{\mathbf{N}} H^c(t, \mathbf{N}^*, \mathbf{\Phi}^*(t), \mathbf{\Psi}^*(t), \mathbf{P}^c(t)) \quad \forall\ c$$

$$\mathbf{P}^c(t_f) = \mathbf{0} \qquad \forall\ c$$

$$\frac{\partial L^{c*}}{\partial Q^c_{[sd]}} = 0 \;\Rightarrow \phi^{c*}_{o[sd]}(t) + \sum_{\pi[sd]} \phi^{c*}_{\pi[sd]}(t) = 1 \qquad \forall\ [sd],\ c$$

$$\frac{\partial L^{c*}}{\partial Q^c_{[s.]}} = 0 \;\Rightarrow \sum_{[.d]} \psi^{c*}_{[sd]}(t) = 1 \quad \forall\ [s.],\ c$$

$$\phi^{c*}_{o[sd]}(t),\ \phi^{c*}_{\pi[sd]}(t) \geq 0 \qquad \forall\ \pi[sd],\ [sd],\ c$$

$$\psi^{c*}_{[sd]}(t) \geq 0 \qquad \forall\ [.d],\ [s.],\ c.\ \square$$

Theorem 3:

Consider the dynamic resource sharing problem in multimedia networks with C competing traffic types, with fixed initial time t_0 and final time t_f. Let for each traffic type c, $g^c(t, \mathbf{N}, \mathbf{\Phi}, \mathbf{\Psi})$, $\mathbf{f}(t, \mathbf{N}, \mathbf{\Phi}, \mathbf{\Psi})$, are continuously differentiable with respect to $(\mathbf{N}, \mathbf{\Phi}, \mathbf{\Psi}) \in (\mathbf{R}^n, \mathbf{AR}, \mathbf{LS})$, $\forall\ t \in [t_0, t_f]$. If $(\hat{\mathbf{\Phi}}^*(t, \mathbf{N}, \mathbf{N}_0), \hat{\mathbf{\Psi}}^*(t, \mathbf{N}, \mathbf{N}_0)) = (\mathbf{\Phi}^*(t), \mathbf{\Psi}^*(t)) \in (\mathbf{AR}, \mathbf{LS})$ is a closed-loop memoryless Nash equilibrium such that $(\hat{\mathbf{\Phi}}^{c*}(t, \mathbf{N}, \mathbf{N}_0), \hat{\mathbf{\Phi}}^{c*}(t, \mathbf{N}, \mathbf{N}_0))$ is continuously differentiable with respect to $\mathbf{N} \in \mathbf{R}^n$, $\forall\ c,\ t \in [t_0, t_f]$ and $\{\mathbf{N}^*(t),\ t \in [t_0, t_f]\}$ is the corresponding state trajectory, then $\exists\ \mathbf{P}^c(t) : [t_0, t_f] \rightarrow \mathbf{R}^n$, $\forall\ c$, continuous and piecewise continuously differentiable vector functions, such that $\forall\ t \in [t_0, t_f]$:

$$
\begin{aligned}
&\dot{\mathbf{N}}^*(t) = \mathbf{f}(t, \mathbf{N}^*(t), \mathbf{\Phi}^*(t), \mathbf{\Psi}^*(t)) \\
&\mathbf{N}^*(t_0) = \mathbf{N}_0 \\
&\left[\frac{\partial H^{c*}}{\partial \phi^c_{o[sd]}} - Q^c_{[sd]}(t)\right] * \phi^{c*}_{o[sd]}(t) = 0 \quad && \forall\ [sd],\ c \\
&\left[\frac{\partial H^{c*}}{\partial \phi^c_{\pi[sd]}} - Q^c_{[sd]}(t)\right] * \phi^{c*}_{\pi[sd]}(t) = 0 \quad && \forall\ \pi[sd],\ [sd], c \\
&\left[\frac{\partial H^{c*}}{\partial \psi^c_{[sd]}} - Q^c_{[s.]}(t)\right] * \psi^{c*}_{[sd]}(t) = 0 \quad && \forall\ [.d],\ [s.],\ c \\
&\frac{\partial H^{c*}}{\partial \phi^c_{o[sd]}} - Q^c_{[sd]}(t) \geq 0 && \forall\ [sd],\ c \\
&\frac{\partial H^{c*}}{\partial \phi^c_{\pi[sd]}} - Q^c_{[sd]}(t) \geq 0 && \forall\ \pi[sd],\ [sd],\ c \\
&\frac{\partial H^{c*}}{\partial \psi^c_{[sd]}} - Q^c_{[s.]}(t) \geq 0 && \forall\ [.d],\ [s.],\ c
\end{aligned}
$$

$$
\begin{aligned}
&\dot{\mathbf{P}}^c(t) = -\nabla_{\mathbf{N}} H^c(t, \mathbf{N}^*, \hat{\mathbf{\Phi}}^*(t, \mathbf{N}^*, \mathbf{N}_0), \hat{\mathbf{\Psi}}^*(t, \mathbf{N}^*, \mathbf{N}_0), \mathbf{P}^c(t)) \quad && \forall\ c \\
&\mathbf{P}^c(t_f) = \mathbf{0} && \forall\ c
\end{aligned}
$$

$$\phi^{c*}_{o[sd]}(t) + \sum_{\pi[sd]\in\mathbf{\Pi}^c_{[sd]}} \phi^{c*}_{\pi[sd]}(t) = 1 \quad \forall\ [sd],\ c$$

$$\sum_{[.d]\in\mathbf{D}^c_{[s.]}} \psi^{c*}_{[sd]}(t) = 1 \quad \forall\ [s.],\ c$$

$$\phi^{c*}_{o[sd]}(t),\ \phi^{c*}_{\pi[sd]}(t) \geq 0 \quad \forall\ \pi[sd],\ [sd],\ c$$

$$\psi^{c*}_{[sd]}(t) \geq 0 \quad \forall\ [.d],\ [s.],\ c$$

Proof: The proof is similar to that for the open-loop solution.□

The following Theorems are easily derived:

Theorem 4: *Admission Control*
Traffic is rejected from the network only if its first derivative of its Hamiltonian w.r.t. its rejection fraction is less than all first derivatives of its Hamiltonian w.r.t. its path routing fractions to its destination:

$\forall c,\ [sd]$:

$$\phi^{c*}_{o[sd]}(t) > 0 \ \ only \ \ if \qquad \frac{\partial H^{c*}}{\partial \phi^c_{o[sd]}} = \min\{\frac{\partial H^{c*}}{\partial \phi^c_{o[sd]}}, \min_{p[sd]}\{\frac{\partial H^{c*}}{\partial \phi^c_{p[sd]}}\}\}$$

$$\phi^{c*}_{o[sd]} = 0 \qquad o.w.$$

$$\phi^{c*}_{o[sd]}(t) + \sum_{\pi[sd]} \phi^{c*}_{\pi[sd]}(t) = 1$$

and satisfies the partial differential vectors for the state (1), (2) and the costate (3), (4) variables.

Theorem 5: *Routing*
There must be traffic only on paths with minimum first derivative of its Hamiltonian w.r.t. its routing fractions and which are less than its first derivative of its Hamiltonian w.r.t. its rejection fraction:

$\forall c,\ [sd],\ \pi[sd]$:

$$\phi^{c*}_{\pi[sd]}(t) > 0 \ \ only \ \ if \qquad \frac{\partial H^{c*}}{\partial \phi^c_{\pi[sd]}} = \min\{\frac{\partial H^{c*}}{\partial \phi^c_{o[sd]}}, \min_{p[sd]}\{\frac{\partial H^{c*}}{\partial \phi^c_{p[sd]}}\}\}$$

$$\phi^{c*}_{\pi[sd]}(t) = 0 \qquad o.w.$$

$$\phi^{c*}_{o[sd]}(t) + \sum_{\pi[sd]} \phi^{c*}_{\pi[sd]}(t) = 1$$

and satisfies the partial differential vectors for the state (1), (2) and the costate (3), (4) variables.

Theorem 6: *Load Sharing*
For each source, there must be traffic only to destinations whose first derivative of its Hamiltonian w.r.t. its load sharing fractions are minimum:

$\forall c,\ [sd]$:

$$\psi^{c*}_{[sd]}(t) > 0 \quad only \quad if \qquad \frac{\partial H^{c*}}{\partial \psi^{c}_{[sd]}} = \min_{[sd']}\{\frac{\partial H^{c*}}{\partial \psi^{c}_{[sd']}}\}$$

$$\psi^{c*}_{[sd]}(t) = 0 \qquad\qquad o.w.$$

$$\sum_{[.d]} \psi^{c*}_{[sd]}(t) = 1$$

and satisfies the partial differential vectors for the state (1), (2) and the costate (3), (4) variables.

So, in this section we have formulated and solved the dynamic resource sharing problem in multimedia networks as a dynamic Nash game among multiple competing traffic types.

4. CONCLUSIONS

In this paper, we propose a game-theoretic approach to the dynamic resource sharing problems in multimedia networks. First, we model the load sharing, routing and admission control mechanisms on the path flow space. Then we introduce dynamic queueing models to describe the dynamic evolution of the number of packets at every network resource. Subsequently, we formulate the dynamic problem as a dynamic Nash game and give the non-cooperative equilibrium conditions. Each decision-maker should allocate its traffic only on its minimum marginal Hamiltonian paths with respect to the load sharing, routing and admission controls.

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